

**Fox Valley Math League**  
**Meet 3 – November 30, 2009**  
**Manitowoc Lincoln High School**



Score

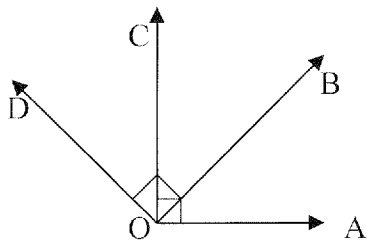
**Event 1**  
 (NO calculators)

**Student Name** *(print)* \_\_\_\_\_ **School Name** \_\_\_\_\_ **Team #** \_\_\_\_\_

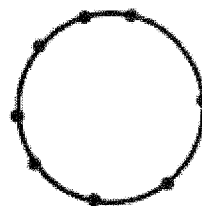
*Every answer must be exact and completely simplified.*

1. Jeff, Gareth, and Ina all share the same birthday. Gareth is one year older than Jeff, and Ina is two years older than Gareth. This year the sum of their ages is 118. How old is Gareth? 1. \_\_\_\_\_

2. In the diagram shown,  $\overline{OA} \perp \overline{OC}$  and  $\overline{OB} \perp \overline{OD}$ . If  $m\angle AOD = 4(m\angle BOC)$ , what is the  $m\angle AOD$ ? 2. \_\_\_\_\_



3. 8 points are placed around the circumference of a circle. How many quadrilaterals can be drawn connecting any four of these points? 3. \_\_\_\_\_



4. Solve. 4. \_\_\_\_\_

$$\left(\frac{1}{25}\right)^x (125)^{x^2} = (125)^x \left(\frac{1}{25}\right)$$

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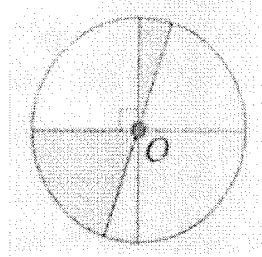
Score

**Event 2**  
(NO calculators)

**Student Name** *(print)* \_\_\_\_\_ **School Name** \_\_\_\_\_ **Team #** \_\_\_\_\_

*Every answer must be exact and completely simplified.*

1. If two of the three diameters shown in the figure are perpendicular, what percent of the circle is shaded? 1. \_\_\_\_\_



2. In a well-shuffled 52-card deck, half the cards are red and half are black. If the number of red cards in the top half is added to the number of black cards in the bottom half, the sum is 30. How many red cards are in the top half? 2. \_\_\_\_\_

3. Solve.  $\frac{5}{x^2 - 4} \geq 1$  3. \_\_\_\_\_

4. Let  $f(x) = \sin(2x)$  and let  $h$  be the smallest positive number so that the graph of  $y = f(x-h)$  will be symmetric about the y-axis. Find  $\cos(h)$  4. \_\_\_\_\_

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**Event 3**  
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**Student Name** *(print)* \_\_\_\_\_ **School Name** \_\_\_\_\_ **Team #** \_\_\_\_\_

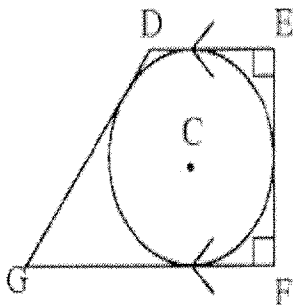
*Every answer must be exact and completely simplified.*

1. If  $x + 2y = 84 = 2x + y$ , what is the value of  $x + y$ ? 1. \_\_\_\_\_

2. If  $r(x)$  means the opposite-reciprocal of  $x$ , what is the value of  $x$  which satisfies  
 $r(x) = r(2) + r(3) + r(4)$  2. \_\_\_\_\_

3. Find the value of  $m$  such that  $(x-2)$  is a factor of  $x^4 + mx^2 - 8x - 3$ . 3. \_\_\_\_\_

4. A trapezoid, DEFG, is circumscribed about a circle that has center C and radius 2. The shorter of the two parallel sides, DE, has length 3 and angles DEF and EFG are right angles. Determine the area of the trapezoid. 4. \_\_\_\_\_



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Score

**Event 4**  
(calculators OK)

**Student Name** *(print)* \_\_\_\_\_ **School Name** \_\_\_\_\_ **Team #** \_\_\_\_\_

*Every answer must be exact and completely simplified OR correctly rounded to the nearest **HUNDREDTH**.*

1. The average of 10 numbers is 63. The average of 6 of these numbers is 57. 1. \_\_\_\_\_  
What is the average of the other 4?

2. Two vertical trees, of heights 10 and 14 meters, are opposite each other, one on 2. \_\_\_\_\_  
each side of a flat road. The distance between the bases of the trees is 16 meters.  
A bird sits on top of each tree. Both sight a worm somewhere between the bases of  
the two trees. At the same time, and with equal speeds, they both dive directly for the  
worm, reaching it simultaneously. What is the distance from the worm to the foot of  
the shorter tree?

3. The sequence of numbers  $t_1, t_2, t_3, \dots$  is defined by  $t_1 = 2$  and  $t_{n+1} = \frac{t_n - 1}{t_n + 1}$ , 3. \_\_\_\_\_  
for every positive integer  $n$ . Determine the *exact* value of  $t_{999}$ .

4. You purchase a DVD drive for your laptop computer. Assume that 65% of the 4. \_\_\_\_\_  
drives are made outside the United States. Of the U.S.-made drives, 4% are defective;  
of the foreign-made drives, 6% are defective. If your drive is defective, determine  
the probability, as a percent, that the drive you purchased is made in the United States.

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**Team Event**  
(calculators)

**Student Name** *(print)* \_\_\_\_\_ **School Name** \_\_\_\_\_ **Team #** \_\_\_\_\_

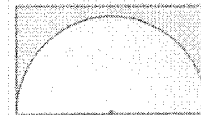
*Every answer must be exact and completely simplified OR correctly rounded to the nearest **HUNDREDTH**.*

1. Two identical triangles each have an area of 24. Their vertices are determined by the intersection of the lines with equations  $y = -4$ ,  $x = 0$ ,  $y = -\frac{3}{4}x + b$ . 1. \_\_\_\_\_

Determine the two possible values for b.

2. Assume that you and 15 of your friends have formed a company called Net-Media, an Internet music and video provider. A committee consisting of a president, a vice president and then a three-member executive board will govern the company. In how many different ways can this committee be formed? 2. \_\_\_\_\_

3. The area of the shaded portion of the rectangle shown is  $6\pi$ , and the area of the semicircle is  $18\pi$ . What is the *exact* perimeter of the rectangle? 3. \_\_\_\_\_



4. Find the sum of all the digits in the number  $(10^{2009} - 2009)$ . 4. \_\_\_\_\_

5. Find the largest value of  $n$  for which  $2^n$  will be a factor of  $20!$  (twenty factorial). 5. \_\_\_\_\_

6. Simplify  $\left(\frac{-1+i\sqrt{3}}{2}\right)^6 + \left(\frac{-2-i\sqrt{2}}{2}\right)^6$  to the form  $a + bi$ . 6. \_\_\_\_\_

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**Event 1**  
 (NO calculators)

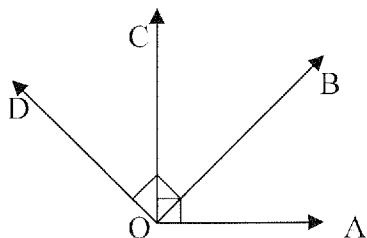
**Student Name** *(print)* \_\_\_\_\_ **School Name** \_\_\_\_\_ **Team #** \_\_\_\_\_

*Every answer must be exact and completely simplified.*

1. Jeff, Gareth, and Ina all share the same birthday. Gareth is one year older than Jeff, and Ina is two years older than Gareth. This year the sum of their ages is 118. How old is Gareth? 1. 39

$$\begin{aligned}
 J + G + I &= 118 \\
 J = G - 1 & \quad (G - 1) + G + (G + 2) = 118 \\
 I = G + 2 & \quad 3G = 117 \\
 G &= 39
 \end{aligned}$$

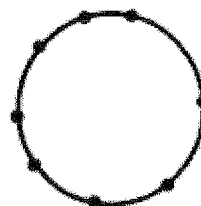
2. In the diagram shown,  $\overline{OA} \perp \overline{OC}$  and  $\overline{OB} \perp \overline{OD}$ . If  $m\angle AOD = 4(m\angle BOC)$ , what is the  $m\angle AOD$ ? 2. 144°



$$\begin{aligned}
 2(90 - m\angle BOC) + m\angle BOC &= m\angle AOD \\
 2(90 - m\angle BOC) + m\angle BOC &= 4(m\angle BOC) \\
 180 - m\angle BOC &= 4(m\angle BOC) & \quad m\angle AOD = 36 + 54 + 54 = 144 \\
 180 &= 5(m\angle BOC) \\
 m\angle BOC &= 36
 \end{aligned}$$

3. 8 points are placed around the circumference of a circle. How many quadrilaterals can be drawn connecting any four of these points? 3. 70

$$\binom{8}{4} = \frac{8!}{4!4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7 \cdot 2 \cdot 5 = 70$$



$$x = \frac{2}{3}, x = 1$$

4. Solve.

$$\left(\frac{1}{25}\right)^x (125)^{x^2} = (125)^x \left(\frac{1}{25}\right)$$

$$\begin{aligned}
 (5^{-2})^x (5^3)^{x^2} &= (5^3)^x (5^{-2}) \\
 5^{-2x+3x^2} &= 5^{3x-2} \\
 3x^2 - 2x &= 3x - 2 \\
 3x^2 - 5x + 2 &= 0 \\
 (3x - 2)(x - 1) &= 0 \\
 x &= \frac{2}{3}, x = 1
 \end{aligned}$$

4. \_\_\_\_\_

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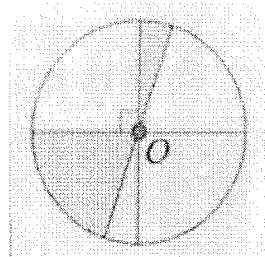
Score

**Event 2**  
 (NO calculators)

**Student Name** *(print)* \_\_\_\_\_ **School Name** \_\_\_\_\_ **Team #** \_\_\_\_\_

*Every answer must be exact and completely simplified.*

1. If two of the three diameters shown in the figure are perpendicular, what percent of the circle is shaded? 1. 25%



2. In a well-shuffled 52-card deck, half the cards are red and half are black. If the number of red cards in the top half is added to the number of black cards in the bottom half, the sum is 30. How many red cards are in the top half? 2. 15

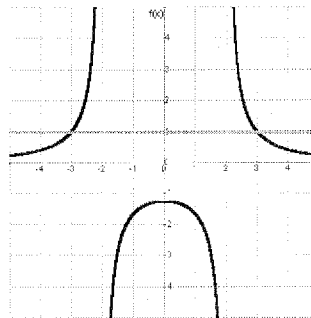
Since there are equal numbers of cards of both colors, there are as many red cards in the top half as there are black cards in the bottom half. Hence the number of red cards in the top half is  $30/2 = 15$

3. Solve.

$$\frac{5}{x^2 - 4} \geq 1$$

$$\begin{aligned} 5 &= x^2 - 4 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

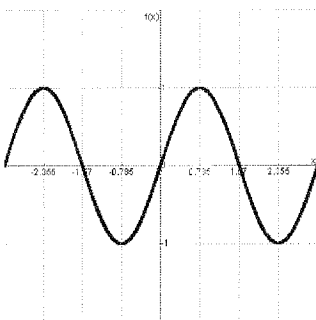
$$\begin{aligned} -3 &\leq x < -2 \\ 2 &< x \leq 3 \end{aligned}$$



3.  $[-3, -2) \cup (2, 3]$   
 OR  
 $-3 \leq x < -2$   
 $2 < x \leq 3$

4. Let  $f(x) = \sin(2x)$  and let  $h$  be the smallest positive number so that the graph of  $y = f(x-h)$  will be symmetric about the y-axis. Find  $\cos(h)$

4.  $\frac{1}{\sqrt{2}}$  OR  $\frac{\sqrt{2}}{2}$



Since  $h$  is positive, the graph of  $f(x-h)$  will shift to the right. To make it symmetrical, shift the graph one-quarter period to the right or  $\pi/4$ .

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

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Score

**Event 3**

(NO calculators)

Student Name *(print)* \_\_\_\_\_ School Name \_\_\_\_\_ Team # \_\_\_\_\_

Every answer must be exact and completely simplified.

1. If  $x + 2y = 84 = 2x + y$ , what is the value of  $x + y$ ? 1. 56

$$\begin{array}{r} x + 2y = 84 \\ 2x + y = 84 \\ \hline 3x + 3y = 168 \\ 3(x + y) = 168 \\ x + y = 56 \end{array}$$

2. If  $r(x)$  means the opposite-reciprocal of  $x$ , what is the value of  $x$  which satisfies 2.  $\frac{12}{13}$

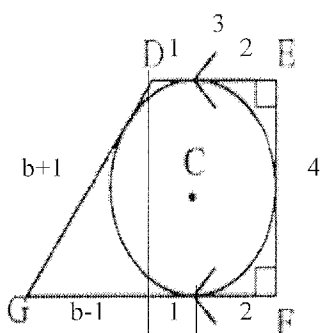
$$\begin{aligned} r(x) &= r(2) + r(3) + r(4) & r(x) &= -\frac{13}{12} \Rightarrow x = \frac{12}{13} \\ &= -\frac{1}{2} - \frac{1}{3} - \frac{1}{4} \\ &= -\frac{6}{12} - \frac{4}{12} - \frac{3}{12} \\ &= -\frac{13}{12} \end{aligned}$$

3. Find the value of  $m$  such that  $(x-2)$  is a factor of  $x^4 + mx^2 - 8x - 3$ . 3.  $m = \frac{3}{4} = 0.75$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & m & -8 & -3 \\ & & 2 & 4 & 8+2m & 4m \\ \hline & 1 & 2 & 4+m & 2m & 4m-3 \end{array}$$

$$\begin{aligned} 4m - 3 &= 0 \\ 4m &= 3 \\ m &= \frac{3}{4} = 0.75 \end{aligned}$$

4. A trapezoid, DEFG, is circumscribed about a circle that has center C and radius 2. The shorter of the two parallel sides, DE, has length 3 and angles DEF and EFG are right angles. Determine the area of the trapezoid. 4. 18



$$\begin{aligned} (b-1)^2 + 4^2 &= (b+1)^2 \\ b^2 - 2b + 1 + 16 &= b^2 + 2b + 1 \\ 16 &= 4b \\ b &= 4 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}(3+6)(4) \\ A &= (9)(2) \\ A &= 18 \end{aligned}$$

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**Event 4**  
 (calculators OK)

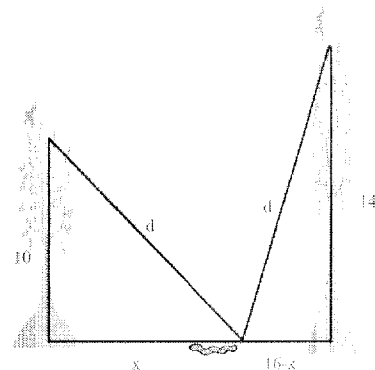
**Student Name** (print) \_\_\_\_\_ **School Name** \_\_\_\_\_ **Team #** \_\_\_\_\_

Every answer must be exact and completely simplified OR correctly rounded to the nearest **HUNDREDTH**.

1. The average of 10 numbers is 63. The average of 6 of these numbers is 57. 1. 72  
 What is the average of the other 4?

$$\begin{aligned} S_{10} &= (10)(63) = 630 & m_4 &= \frac{288}{4} = 72 \\ S_6 &= (6)(57) = 342 \\ S_4 &= 630 - 342 = 288 \end{aligned}$$

2. Two vertical trees, of heights 10 and 14 meters, are opposite each other, one on each side of a flat road. The distance between the bases of the trees is 16 meters. A bird sits on top of each tree. Both sight a worm somewhere between the bases of the two trees. At the same time, and with equal speeds, they both dive directly for the worm, reaching it simultaneously. What is the distance from the worm to the foot of the shorter tree?



$$\begin{aligned} x^2 + 10^2 &= (16 - x)^2 + 14^2 \\ x^2 + 100 &= 256 - 32x + x^2 + 196 \\ 100 &= 452 - 32x \\ 32x &= 352 \\ x &= 11 \end{aligned}$$

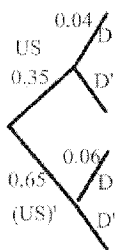
3. The sequence of numbers  $t_1, t_2, t_3, \dots$  is defined by  $t_1 = 2$  and  $t_{n+1} = \frac{t_n - 1}{t_n + 1}$ , 3.  $-\frac{1}{2}$  or  $-0.5$

for every positive integer  $n$ . Determine the *exact* value of  $t_{999}$ .

$$\begin{aligned} t_1 &= 2 \\ t_2 &= \frac{t_1 - 1}{t_1 + 1} = \frac{2 - 1}{2 + 1} = \frac{1}{3} \\ t_3 &= \frac{t_2 - 1}{t_2 + 1} = \frac{\frac{1}{3} - 1}{\frac{1}{3} + 1} = -\frac{1}{2} \\ t_4 &= \frac{t_3 - 1}{t_3 + 1} = \frac{-\frac{1}{2} - 1}{-\frac{1}{2} + 1} = -3 \\ t_5 &= \frac{t_4 - 1}{t_4 + 1} = \frac{-3 - 1}{-3 + 1} = 2 \end{aligned}$$

Since a term in the sequence depends only on the previous one, then the sequence will cycle with a period of 4. Thus,  $t_1 = t_5 = \dots = t_{997} = t_{1001} = 2$ . Therefore,  $t_{998} = \frac{1}{3}$  and  $t_{999} = -\frac{1}{2}$ .

4. You purchase a DVD drive for your laptop computer. Assume that 65% of the drives are made outside the United States. Of the U.S.-made drives, 4% are defective; of the foreign-made drives, 6% are defective. If your drive is defective, determine the probability, as a percent, that the drive you purchased is made in the United States. 4. 26.42%



$$P(US | D) = \frac{P(US \cap D)}{P(D)} = \frac{(0.35)(0.04)}{(0.65)(0.06) + (0.35)(0.04)} \approx 0.26415$$

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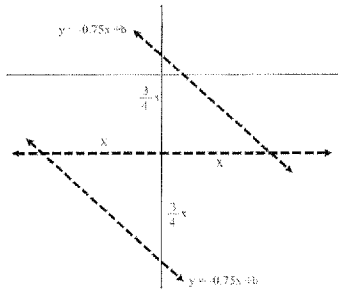
Score

**Team Event**

(calculators)

Every answer must be exact and completely simplified OR correctly rounded to the nearest **HUNDREDTH**.

1. Two identical triangles each have an area of 24. Their vertices are determined by the intersection of the lines with equations  $y = -4$ ,  $x = 0$ ,  $y = -\frac{3}{4}x + b$ . 1. -10 or 2



Determine the two possible values for b.

$$A = \frac{1}{2}(x)\left(\frac{3}{4}x\right) = 24$$

$$\frac{3}{8}x^2 = 24$$

$$x^2 = 64$$

$$x = \pm 8$$

$$y = \pm 6$$

$$b = -4 \pm 6$$

$$b = -10 \quad \text{or} \quad b = 2$$

2. Assume that you and 15 of your friends have formed a company called Net-Media, an Internet music and video provider. A committee consisting of a president, a vice president and then a three-member executive board will govern the company. In how many different ways can this committee be formed? 2. 87360

$$({}_{16}P_2) \cdot ({}_{14}C_3) = 87360$$

3. The area of the shaded portion of the rectangle shown is  $6\pi$ , and the area of the semicircle is  $18\pi$ . What is the *exact* perimeter of the rectangle? 3. (must be exact)

$$\frac{1}{2}\pi r^2 = 18\pi$$

$$r^2 = 36$$

$$r = 6$$

$$l = 12$$

$$12h - 18\pi = 6\pi$$

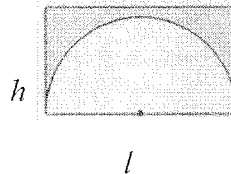
$$12h = 24\pi$$

$$h = 2\pi$$

$$P = 2l + 2h$$

$$P = 2(12) + 2(2\pi)$$

$$P = 24 + 4\pi$$



$$\frac{24 + 4\pi}{1}$$

4. Find the sum of all the digits in the number  $(10^{2009} - 2009)$ . 4. 18071

$$10^{2009} - 2009$$

will have 2007 9s, a 7, and a 1

$$2007 \cdot 9 + 7 + 1 = 18071$$

5. Find the largest value of  $n$  for which  $2^n$  will be a factor of  $20!$  (twenty factorial). 5. 18

Even factors of  $20!$  are 20, 18, 16, 15, 14, 12, 10, 8, 6, 4, 2. The total number of factors of 2 in those numbers is 18.

6. Simplify  $\left(\frac{-1+i\sqrt{3}}{2}\right)^6 + \left(\frac{-2-i\sqrt{2}}{2}\right)^6$  to the form  $a + bi$ . 6. (2+i) or (2+1i)

$$\left(\cos \frac{2\pi}{3} + i \sin \frac{\pi}{3}\right)^6 + \left(\cos \pi + i \sin \frac{-\pi}{4}\right)^6$$

$$\left(\cos\left(6\left(\frac{2\pi}{3}\right)\right) + i \sin\left(6\left(\frac{\pi}{3}\right)\right)\right) + \left(\cos 6\pi + i \sin\left(6\left(\frac{-\pi}{4}\right)\right)\right)$$

$$(\cos 4\pi + i \sin 2\pi) + \left(\cos 6\pi + i \sin \frac{-3\pi}{2}\right) = (1 + 0i) + (1 + 1i) = 2 + i$$